DCS/CSCI 2350: Social & Economic Networks

Games and game theory: A brief introduction

Reading: Ch. 6 of EK

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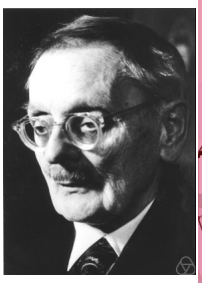
Game Theory

- "Game"

  Ernst Zermelo (1913):
  In any chess game that does not end in a draw, a player has a winning strategy



- Mathematical theory of strategic decision making
  John von Neumann (1944)

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Applications

- Application: market equilibria
  - Predict where the market is heading to
- Mechanism design and auctions
  - Google and Yahoo apply game-theoretic techniques
    - Keyword search auction
  - Spectrum allocation among wireless companies

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Applications

Understanding the Internet: “Selfish routing” is a constant-factor off from optimal

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Applications

- Load balancing and resource allocation
- p2p and file sharing systems
- Cryptography and security
- Social and economic networks, etc.

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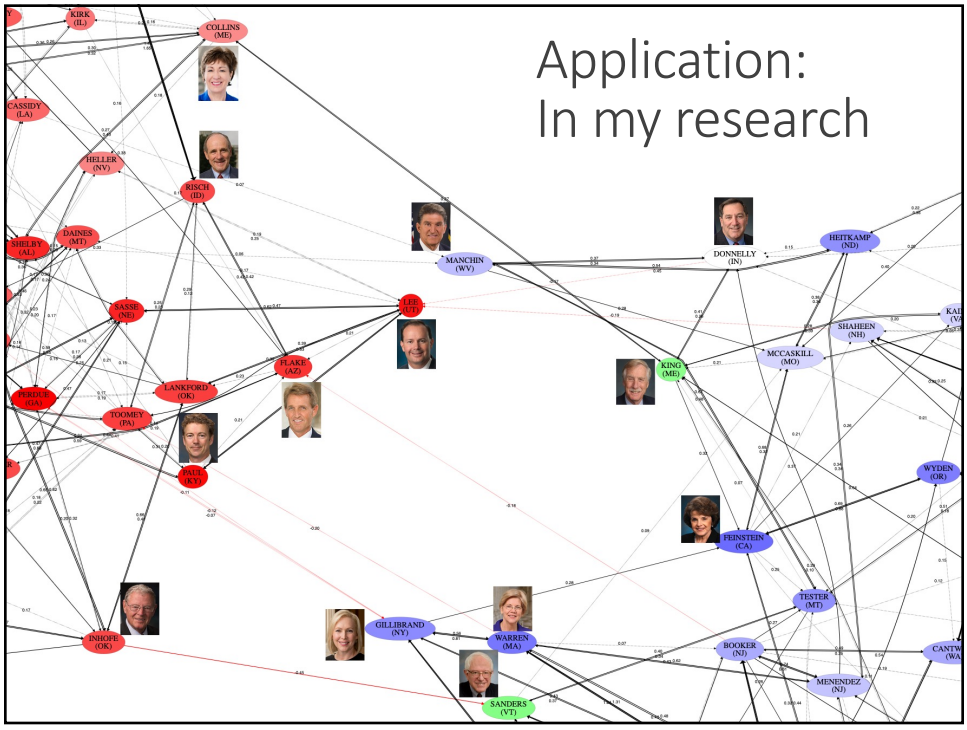

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The Power Of Context in Networks:
Ideal Point Models with Social Interactions

AAMAS 2018, Sweden

With Tucker Gordon'17

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Spheres Of Legislation: Polarization And Most Influential Nodes In Behavioral Context

Complex Networks 2019, Portugal.

With Andrew Phillips'19 &
Luca Ostertag-hill'20

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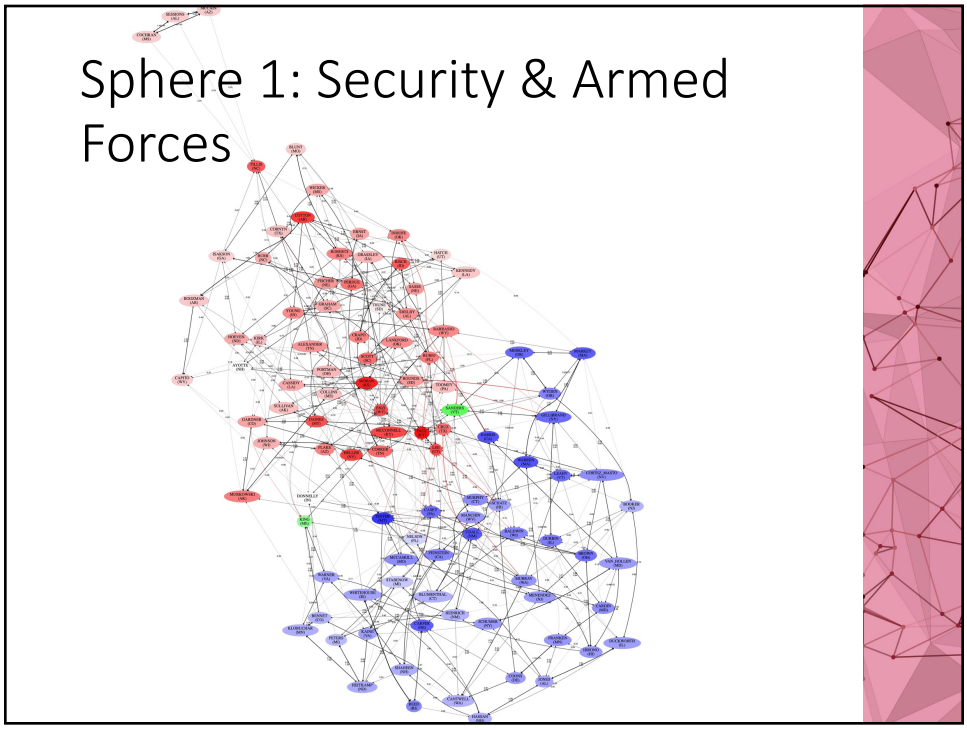

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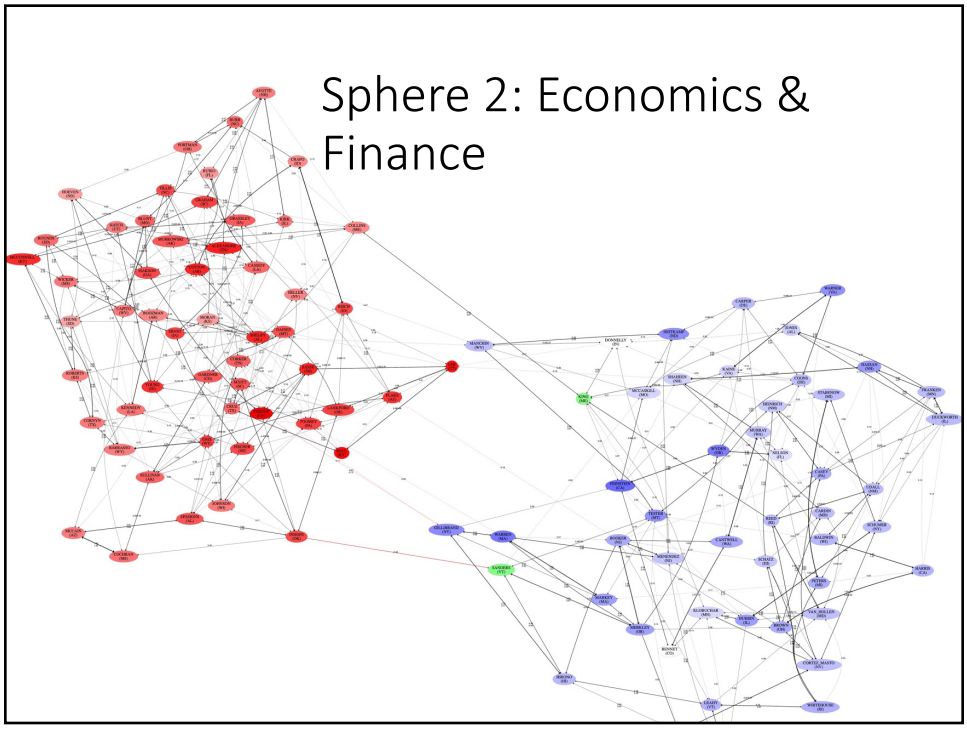

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Discussion:
- game
- best response
- dominant strategy
- Nash equilibrium (NE)
- pure-strategy NE (PSNE)

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Example: Split or Steal

https://www.youtube.com/watch?v=yM38mRHY150

- Rules of the game
- Outcome

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Game model of split or steal

- One-shot game (simultaneous move)
- 3 components
  - Players
  - Strategies/actions
  - Payoffs

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Lucy

Tony	Payoff matrix	Split	Steal
	Split	$33K, $33K	Frust., $66K
	Steal	$66K, Frust.	$0, $0

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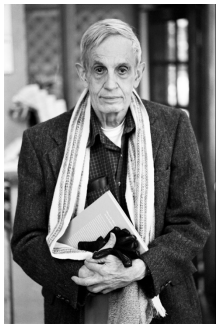
Why did they end up with 0?

Tony \ Lucy	Split	Steal
Split	$33K, $33K	Frust., $66K
Steal	$66K, Frust.	$0, $0

Nash Equilibrium
Everyone plays their best response to others simultaneously



John F. Nash
Nobel Prize, 1994

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Best response

- Best strategy of a player, given the other players' strategies
- Always exists!

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(Strictly/weakly) dominant strategy

- A strategy of a player that is (strictly/weakly) better than any of their other strategies, no matter what the other players do
- Does not always exist

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Famous example: prisoner's dilemma

		Suspect 2	
	Payoff matrix	Not Confess	Confess
Suspect 1	Not Confess	-1, -1	-10, 0
	Confess	0, -10	-4, -4

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Drug usage in cycling

Cyclist 2

Cyclist 1	Payoff matrix	No Drugs	Drugs
	No Drugs	3, 3	1, 4
	Drugs	4, 1	2, 2

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Example

(Weakly) dominant strategy

Neither is a dominant strategy

Column Player

Row Player	Payoff matrix	L	R
	U	10, 50	5, 0
	D	0, 0	5, 10

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Checkpoint

- What is the difference between a dominant strategy and a best response?
- What is the difference between weakly and strictly dominant strategies? Will a player always have one?

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Discussion:
- game
- best response
- dominant strategy
- Nash equilibrium (NE)
- pure-strategy NE (PSNE)

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Nash equilibrium (NE)

- A joint strategy (one strategy/player) s.t. every player plays their best response to others simultaneously
- (Equiv.) A joint strategy s.t. no player gains by deviating unilaterally
  - Useful for checking whether a joint strategy is a NE

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Pure-strategy Nash equilibrium (PSNE)

- Players do not use any probability in choosing strategies as they do in "mixed-strategy"
- Every player plays their best response to others simultaneously

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Checkpoint

- What is the difference between best response and PSNE?
- Is there a connection between dominant strategy and PSNE?

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Quiz

- Watch the following clip from the movie *a Beautiful Mind* portraying Nash's discovery of NE
  https://www.youtube.com/watch?v=LJS7Igvk6ZM
- Is this actually a Nash equilibrium?
  - Detailed answer: A blog post (also posted on the class website)

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Misconceptions

- Equilibrium signifies a tie/draw/balance
- Equilibrium outcome is the best possible outcome for all players (*A Beautiful Mind*)
- Self-interested players want to hurt each other

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Questions

- Does NE always exist? (Answer later ...)
- If it exists, is it unique?

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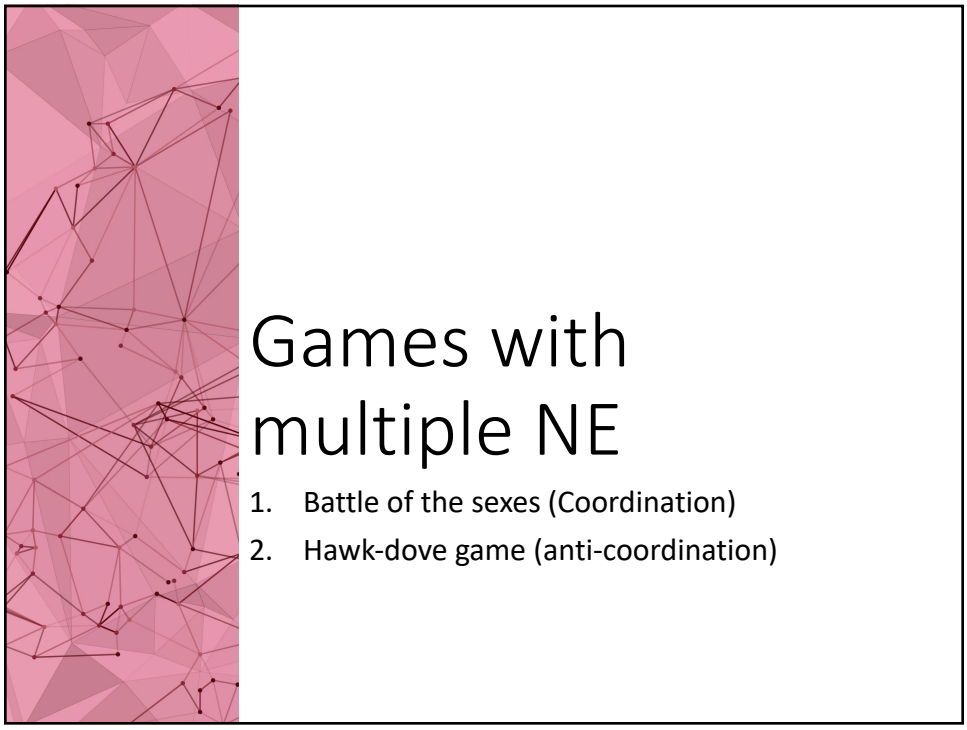
Games with multiple NE
1. Battle of the sexes (Coordination)
2. Hawk-dove game (anti-coordination)

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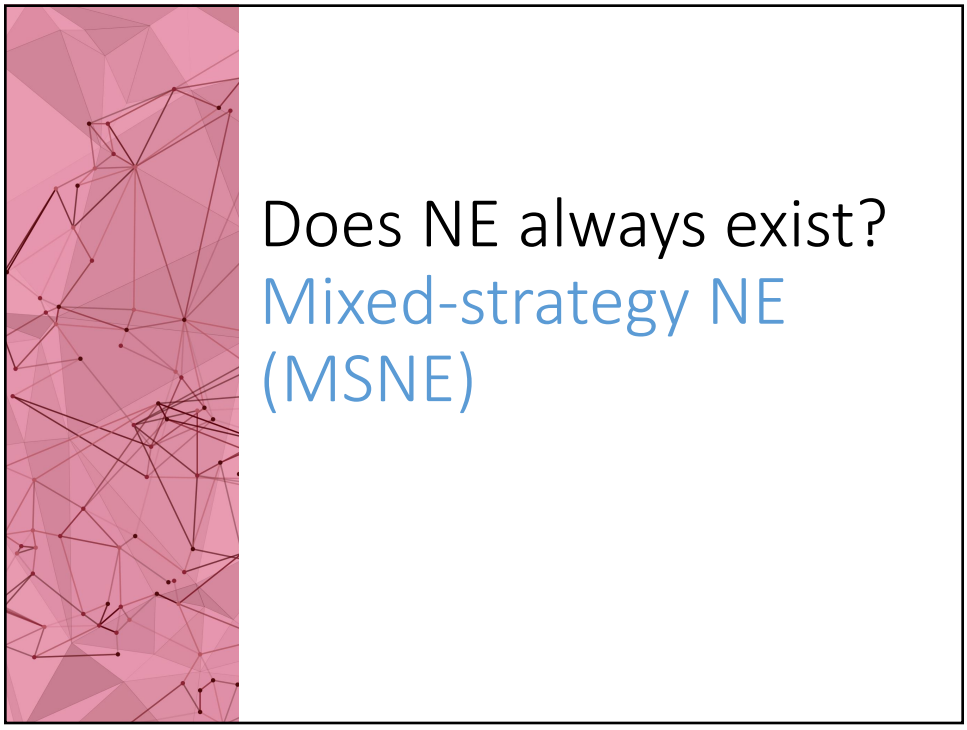
Does NE always exist?
Mixed-strategy NE (MSNE)

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Penalty kick game

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Penalty kick game (continued)

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Penalty kick game (continued)

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Penalty kick game (real-world)

- *"Professionals Play Minimax"*- Ignacio Palacios-Huerta

Equilibrium probabilities (computed by solving equations) match real-world probabilities from data!

		Goalkeeper: Left (0.42)	Goalkeeper: Right (0.58)
Shooter	Left (0.38)	0.58, 0.42	0.95, 0.05
Shooter	Right (0.62)	0.93, 0.07	0.70, 0.30

From real-world data

https://bleacherreport.com/articles/755195-champions-league-08-analyzing-one-of-the-most-iconic-shootouts-in-history

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Von Neumann's Theorem (1928)

- Every finite 2-person zero-sum game has a mixed equilibrium

John von Neumann (1903 – 1957)

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Theorem of Nash (1950)

Every finite game has an equilibrium in mixed strategies

John F. Nash (1928 – 2015)
Nobel Prize, 1994

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Key take-away messages

- Players act simultaneously, but NE outcome is stable in the sense that there is no incentive for unilateral deviation.
- There is always at least one MSNE (including PSNE). A PSNE is not guaranteed.
- The concept of NE doesn't say **how** NE happens.
- NE is not a balance or tie. It is often times a socially-inefficient outcome.

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